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Decentralized Event-Triggered Control of Networked Systems–Part 2: Containment Control

Teng-Hu Cheng[1], Zhen Kan[1], Justin R. Klotz[1], John M. Shea[2], Warren E. Dixon[1,2]

***Abstract*—A decentralized event-triggered control scheme is developed for the containment control problem. An estimate-based decentralized controller is designed for each agent so that it is only required to communicate with neighboring agents at discrete event times. These events are determined by a decentralized trigger function that only requires local information. Different from conventional strategies, the developed control approach does not require continuous communication with local neighboring follower agents for state feedback, reducing communication bandwidth. The event-triggered approach is facilitated by developing a positive constant lower bound on the inter-event interval, which indicates Zeno behavior is avoided. A Lyapunov-based convergence analysis is provided to indicate asymptotic convergence of the developed strategy.**

I. INTRODUCTION

Various applications can be facilitated through the use of a team of collaborative agents. Ideally, communication required for navigation and control of the agents is minimized to maximize the available bandwidth for other mission objectives (e.g., relaying sensing data). Towards this objective, network control approaches have been developed using strategies that only require local communication (e.g., from one and two hop neighbors), and leader/follower strategies where follower agents have partial feedback information [1]–[4]. For example, the containment control problem considered in this paper focuses on a decentralized strategy in which a subgroup of agents (i.e., followers) must remain within a finite region spanned by another subgroup (i.e., leaders), where the leader states are only communicated to a subset of followers (i.e., followers that have leader neighbors). However, even for the decentralized containment control problem, most existing solutions (cf. [4]–[8]) require continuous state feedback to be communicated from neighboring agents for decentralized implementation.

To reduce inter-agent communication, real-time scheduling methods, called event-triggered approaches (cf. [9], [10]), can be applied on an as-needed basis to reduce continuous state feedback. Typically in event-triggered control, the control task is executed when the ratio of a certain error norm to the state norm exceeds a threshold. As a result, when compared to traditional continuous feedback methods, event-triggered execution yields a minimum inter-event interval.

Motivated by the desire to reduce communication traffic and the controller updates, event-triggered results have been developed for multi-agent systems in [11]–[17]. However, these applications target the same average consensus problem with a leaderless network and require continuous communication with neighboring agents for event detection. Therefore, these event-triggered approaches may not mitigate communication congestion for a large scale network. These results are extended in [18] for a dynamic leader, intermittent communication, and communication-free event detection. However, it is unclear how to directly extend the approaches in [11]–[18] to the containment control problem. This paper develops an approach for containment control without continuous communication.

Similar to the development in [18], this paper develops a decentralized estimator-based event-triggered containment control approach where every follower agent has model-based state estimators dwelling at its neighboring agents as well as itself. These distributed estimators follow the same dynamics as the leader and are synchronized by simultaneous updates at discrete events. Since the follower agents know how far the state estimates are away from its true state, it can communicate its true state to these estimates when necessary, but not vice versa (i.e., neighboring agents have no authority to request updates). This event is generated by a decentralized, estimate-based, event-triggered function. As a result, any follower agent has a decentralized, estimate-based, piecewise continuous controller, which is discontinuous at the event times whenever broadcasting its state to, or receiving estimate updates from, the neighboring agents is required. Therefore, no inter-agent communication is required between any two event times. A lower bounded minimum inter-event interval can be developed, and a convergence analysis shows that the individual agent requires only intermittent communications for asymptotic convergence.

This paper builds on the event-triggered strategy in [18]; however, this paper is focused on the more general containment control problem where multiple leaders are included in the network. In both papers, the dynamics of the agents, communication mechanism, and the proof of minimum inter-event interval are identical; however, the controller design and stability analysis in this paper are unique due to the

[1]Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville FL 32611-6250, USA Email:{tenghu, kanzhen0322, jklotz, wdixon}@ufl.edu
[2]Department of Electrical and Computer Engineering, University of Florida, Gainesville FL 32611-6250, USA Email: jshea@ece.ufl.edu, wdixon@ufl.edu.
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generalization of multiple leaders. While this paper investigates a more general multiple-leader containment control problem, unlike [18], the topology in this paper is assumed to be fixed. It is currently unclear how to leverage the switched topology strategy in [18] to the current containment control result because of the time-varying links between the leaders and followers.

II. Preliminaries

A. Preliminaries

To describe the interaction between follower nodes, an undirected graph $\mathcal{G}_{\mathcal{F}} = (\mathcal{V}_{\mathcal{F}}, \mathcal{E}_{\mathcal{F}})$ is defined, where $\mathcal{V}_{\mathcal{F}} \triangleq \{1, \ldots, F\}$ is the index set of the F follower nodes, and $\mathcal{E}_{\mathcal{F}} \subseteq \mathcal{V}_{\mathcal{F}} \times \mathcal{V}_{\mathcal{F}}$ is the corresponding edge set. An undirected edge (i, j) is an element of $\mathcal{E}_{\mathcal{F}}$ if $i, j \in \mathcal{V}_{\mathcal{F}}$ can exchange information mutually. Without loss of generality, the undirected graph is assumed to be simple (i.e., $(i, i) \notin \mathcal{E}$, $\forall i \in \mathcal{V}_{\mathcal{F}}$). A path is a sequence of connected edges in a graph. Graph $\mathcal{G}_{\mathcal{F}}$ is connected if there exists a path between any two nodes in $\mathcal{G}_{\mathcal{F}}$. The follower neighbor set $\mathcal{N}_{\mathcal{F}i} \triangleq \{j \in \mathcal{V}_{\mathcal{F}} \mid (j, i) \in \mathcal{E}_{\mathcal{F}}\}$ is a set of follower nodes that can deliver information to agent i. An adjacency matrix $\mathcal{A}_{\mathcal{F}} = [a_{ij}] \in \mathbb{R}^{F\times F}$ of graph $\mathcal{G}_{\mathcal{F}}$ is defined such that $a_{ii} = 0$, $a_{ij} = a_{ji} = 1$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = a_{ji} = 0$ otherwise. The Laplacian matrix of graph $\mathcal{G}_{\mathcal{F}}$ is defined as $\mathcal{L}_1 = [l_{ij}] \in \mathbb{R}^{F\times F}$, where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. To describe the interaction topology of all nodes, a directed graph $\mathcal{G} = (\mathcal{V}_{\mathcal{F}} \cup \mathcal{V}_{\mathcal{L}}, \mathcal{E}_{\mathcal{F}} \cup \mathcal{E}_{\mathcal{L}})$ is defined as a supergraph of $\mathcal{G}_{\mathcal{F}}$ formed by connecting an additional edge $(k, i) \in \mathcal{E}_{\mathcal{L}}$ to $\mathcal{G}_{\mathcal{F}}$ if the leader $k \in \mathcal{V}_{\mathcal{L}}$ communicates information to the follower $i \in \mathcal{V}_{\mathcal{F}}$, where $\mathcal{V}_{\mathcal{L}} \triangleq \{F+1, \ldots, F+L\}$ is the indexed set of the leader nodes, and $\mathcal{E}_{\mathcal{L}} \subseteq \mathcal{V}_{\mathcal{L}} \times \mathcal{V}_{\mathcal{F}}$ is a leader-follower edge set. The leader neighbor set $\mathcal{N}_{\mathcal{L}i} \triangleq \{j \in \mathcal{V}_{\mathcal{L}} \mid (j, i) \in \mathcal{E}_{\mathcal{L}}\}$ is a set of leaders that can deliver information to follower i. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(F+L)\times(F+L)}$ of graph $\mathcal{G}$ is also defined such that $a_{ii} = 0$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}_{\mathcal{F}} \cup \mathcal{E}_{\mathcal{L}}$, and $a_{ij} = 0$ otherwise. Similar to $\mathcal{L}_1$, the Laplacian matrix of graph $\mathcal{G}$ can be expressed as $\mathcal{L} = \begin{bmatrix} \mathcal{L}_{\mathcal{F}} & \mathcal{L}_{\mathcal{L}} \\ 0_{L\times F} & 0_{L\times L} \end{bmatrix}$, where $\mathcal{L}_{\mathcal{L}} \in \mathbb{R}^{F\times L}$, 0 is the zero matrix of defined dimensions, and $\mathcal{L}_{\mathcal{F}} \triangleq \mathcal{L}_1 + D \in \mathbb{R}^{F\times F}$ is a symmetric matrix, where $D = [d_{ij}] \in \mathbb{R}^{F\times F}$ is a diagonal matrix defined such that $d_{ii} = \sum_{l\in\mathcal{V}_{\mathcal{L}}} a_{il}$ and $d_{ij} = 0$ for $i \neq j$.

To facilitate the subsequent analysis, the following lemma from [5] is provided.

Lemma 1. *[5] If graph $\mathcal{G}$ is connected, then the symmetric matrix $\mathcal{L}_{\mathcal{F}}$ is positive definite.*

B. Dynamics

Consider a network system composed of F follower agents and L leader agents, with dynamics

$$\dot{x}_i = Ax_i + Bu_i,\ i \in \mathcal{V}_{\mathcal{F}} \tag{1}$$

$$\dot{x}_i = Ax_i, \qquad i \in \mathcal{V}_{\mathcal{L}} \tag{2}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input of agent i, respectively, $B \in \mathbb{R}^{n\times m}$ is a full column rank matrix, and $A \in \mathbb{R}^{n\times n}$ is a state matrix.

Assumption 1. The dynamics of the agents are controllable, or the pair (A, B) is stabilizable.

Assumption 2. Each follower has directed paths from at least one leader.

III. Development of the Event-triggered Decentralized Controller

The containment control objective is to ensure the states of all the followers converge to the convex hull spanned by the leaders' states, such as [4]

$$\left\| x_{\mathcal{F}} + \left(\mathcal{L}_{\mathcal{F}}^{-1}\mathcal{L}_{\mathcal{L}} \otimes I_n\right) x_{\mathcal{L}} \right\| \to 0 \text{ as } t \to \infty. \tag{3}$$

In this section, an event-triggered based decentralized controller is developed to minimize the inter-agent communication while achieving the containment control objective defined in (3). Different from conventional approaches, event-triggered control methods generate a piecewise continuous control signal, where the discontinuities are due to the state estimate updates. The discrete events are generated from the satisfaction of a triggering condition. The triggering condition in this paper is designed based on insights from the Lyapunov-based state convergence analysis.

A. Controller design

Based on the subsequent convergence analysis, the decentralized event-triggered controller for agent $i \in \mathcal{V}_{\mathcal{F}}$ is designed as

$$u_i = K\hat{z}_i \tag{4}$$

$$\hat{z}_i = \sum_{j\in\mathcal{N}_{\mathcal{F}i}} (\hat{x}_j - \hat{x}_i) + \sum_{j\in\mathcal{N}_{\mathcal{L}i}} (x_j - \hat{x}_i),\ i \in \mathcal{V}_{\mathcal{F}}, \tag{5}$$

where K is the control gain designed as

$$K = B^T P. \tag{6}$$

Based on Assumption 1, $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix that satisfies the following Riccati inequality

$$PA + A^T P - 2\delta_{\min} PBB^T P + \delta_{\min} I_n < 0, \tag{7}$$

where I is an identity matrix with denoted dimension, and $\delta_{\min} \in \mathbb{R}^+$ denotes the minimum eigenvalue of $\mathcal{L}_{\mathcal{F}}$, where $\mathcal{L}_{\mathcal{F}}$ is positive definite based on Assumption 2 and Lemma 1.

In (5), the followers that are connected to the leader can continuously receive information from the leader, and the computation of $\hat{z}_i$ in (5) only requires the estimates of agent i and its neighboring followers' state (i.e., $\hat{x}_{j\in\mathcal{N}_{\mathcal{F}i}}$), instead of using their true states $x_{j\in\mathcal{N}_{\mathcal{F}i}}$ via continuous communication.

The estimate $\hat{x}_j$ in (5) evolves according to the following dynamics

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t), \; j \in \mathcal{N}_{\mathcal{F}i} \cup \{i\}, \; t \in \left[t_k^j, t_{k+1}^j\right), \quad (8)$$

$$\hat{x}_j\left(t_k^j\right) = x_j\left(t_k^j\right), \quad (9)$$

for $k = 0, 1, 2\ldots,$ where $\hat{x}_j$ flows along the leader dynamics during $t \in \left[t_k^j, t_{k+1}^j\right)$ and is updated via x_j communicated from neighboring agent j at its discrete times t_k^j, for $j \in \mathcal{N}_{\mathcal{F}i}$. Although agent $i \in \mathcal{V}_{\mathcal{F}}$ does not communicate the estimate $\hat{x}_i$, agent i maintains $\hat{x}_i$ for implementation in (5). The estimate $\hat{x}_i$ is updated continuously with the dynamics in (8) and discretely at time instances described in (9). Therefore, u_i is a piecewise continuous signal, which has discontinuities when state information is transmitted to, or received from, neighboring agents for estimate updates; otherwise, u_i flows continuously during the inter-event intervals. The generation of the event times will be described in Section III-C.

B. Dynamics of estimate errors

Since x_i follows different dynamics from the estimate $\hat{x}_i$ for $i \in \mathcal{V}_{\mathcal{F}}$, an estimate error $e_i \in \mathbb{R}^n$ characterizing the mismatch is defined as

$$e_i(t) \triangleq \hat{x}_i(t) - x_i(t), \; i \in \mathcal{V}_{\mathcal{F}}, \; t \in \left[t_k^i, t_{k+1}^i\right), \quad (10)$$

where e_i is reset to 0 at the event time t_k^i, $k = 0, 1, 2, \ldots$. Although x_i and $\hat{x}_i$ are both known for agent i, using $\hat{x}_i$ enables agent i to judge how far another $\hat{x}_i$ in its neighboring agent is away from its actual state x_i. Using (1), (4), and (8), the time-derivative of (10) can be expressed as

$$\dot{e}_i = A(\hat{x}_i - x_i) - BK\sum_{j\in\mathcal{N}_{\mathcal{F}i}}(\hat{x}_j - \hat{x}_i) - BK\sum_{j\in\mathcal{N}_{\mathcal{L}i}}(x_j - \hat{x}_i), \; t \in \left[t_k^i, t_{k+1}^i\right),$$

which has a stacked form of

$$\dot{e} = (I_F \otimes A)e + (I_F \otimes BK)\varepsilon + (\mathcal{L}_{\mathcal{F}} \otimes BK)e, \quad (11)$$

where $e \in \mathbb{R}^{nF}$ denotes $e \triangleq \left[e_1^T, e_2^T, \ldots, e_F^T\right]^T$, $\otimes$ denotes the Kronecker product, and $\varepsilon \in \mathbb{R}^{nF}$ is a stacked form of ε_i defined as $\varepsilon \triangleq \left[\varepsilon_1^T, \ldots, \varepsilon_F^T\right]^T$, where $\varepsilon_i \in \mathbb{R}^n$ represents the relative neighboring state tracking error as

$$\varepsilon_i \triangleq \sum_{j\in\mathcal{N}_{\mathcal{F}i}\cup\mathcal{N}_{\mathcal{L}i}}(x_i - x_j), \; i \in \mathcal{V}_{\mathcal{F}}, \quad (12)$$

which has a stacked form

$$\varepsilon = (\mathcal{L}_{\mathcal{F}} \otimes I_n)x_{\mathcal{F}} + (\mathcal{L}_{\mathcal{L}} \otimes I_n)x_{\mathcal{L}}. \quad (13)$$

C. Event-triggered communication mechanism

A follower agent's state estimate is updated whenever communication is triggered by a neighbor's trigger condition or its own trigger condition. Please see Section III-C of [18] for further details on the communication mechanism. The triggered condition is defined in Section IV.

D. Closed-Loop error system

Using (10), a non-implementable form (to facilitate the subsequent analysis) of (4) can be written as

$$u_i(t) = K\sum_{j\in\mathcal{N}_{\mathcal{F}i}\cup\mathcal{N}_{\mathcal{L}i}}\left[(x_j(t) - x_i(t)) + (e_j(t) - e_i(t))\right], \quad (14)$$

where $e_{j\in\mathcal{V}_{\mathcal{L}}} = 0$ due to continuous communication from leaders. Substituting (14) into the open-loop dynamics in (1) yields

$$\dot{x}_i = Ax_i + BK\sum_{j\in\mathcal{N}_{\mathcal{F}i}\cup\mathcal{N}_{\mathcal{L}i}}(x_j(t) - x_i(t)) + BK\sum_{j\in\mathcal{N}_{\mathcal{F}i}\cup\mathcal{N}_{\mathcal{L}i}}(e_j(t) - e_i(t)),$$

or equivalently

$$\dot{x}_{\mathcal{F}} = (I_F \otimes A)x_{\mathcal{F}} - (\mathcal{L}_{\mathcal{F}} \otimes BK)x_{\mathcal{F}} - (\mathcal{L}_{\mathcal{L}} \otimes BK)x_{\mathcal{L}} - (\mathcal{L}_{\mathcal{F}} \otimes BK)e \quad (15)$$

$$\dot{x}_{\mathcal{L}} = (I_F \otimes A)x_{\mathcal{L}}, \quad (16)$$

where $x_{\mathcal{F}} \triangleq \left[x_1^T, \ldots, x_F^T\right]^T \in \mathbb{R}^{nF}$, $x_{\mathcal{L}} \triangleq \left[x_1^T, \ldots, x_L^T\right]^T \in \mathbb{R}^{nL}$ are the stacked states of the follower and leader agents, respectively. Using (15) and (16), the closed-loop error system can be expressed as

$$\begin{aligned}\dot{\varepsilon} &= (\mathcal{L}_{\mathcal{F}} \otimes I_n)\dot{x}_{\mathcal{F}} + (\mathcal{L}_{\mathcal{L}} \otimes I_n)\dot{x}_{\mathcal{L}} \\ &= (\mathcal{L}_{\mathcal{F}} \otimes I_n)\left[(I_F \otimes A)x_{\mathcal{F}} - (\mathcal{L}_{\mathcal{F}} \otimes BK)x_{\mathcal{F}} - (\mathcal{L}_{\mathcal{L}} \otimes BK)x_{\mathcal{L}} - (\mathcal{L}_{\mathcal{F}} \otimes BK)e\right] \\ &\quad + (\mathcal{L}_{\mathcal{L}} \otimes I_n)(I_F \otimes A)x_{\mathcal{L}} \\ &= \left[(I_F \otimes A) - (\mathcal{L}_{\mathcal{F}} \otimes BK)\right]\varepsilon - \left(\mathcal{L}_{\mathcal{F}}^2 \otimes BK\right)e, \quad (17)\end{aligned}$$

where (13) is used.

To facilitate the subsequent convergence analysis, an alternative form of (13) associated with an auxiliary function $\hat{z} \triangleq \left[\hat{z}_1^T, \ldots, \hat{z}_F^T\right]^T \in \mathbb{R}^{nF}$ is developed. Based on (10) and (12), the relative state tracking error can be expressed as

$$\begin{aligned}\varepsilon_i &= \sum_{j\in\mathcal{N}_{\mathcal{F}i}}\left[(\hat{x}_i - e_i) - (\hat{x}_j - e_j)\right] + \sum_{j\in\mathcal{N}_{\mathcal{L}i}}\left[(\hat{x}_i - e_i) - x_j\right] \\ &= -\hat{z}_i - \sum_{j\in\mathcal{N}_{\mathcal{F}i}}(e_i - e_j) - \sum_{j\in\mathcal{N}_{\mathcal{L}i}}e_i, \quad i \in \mathcal{V}_{\mathcal{F}}, \quad (18)\end{aligned}$$

where $\hat{z}_i$ is defined in (5). The stacked form of ε_i in (18) can be expressed as

$$\varepsilon = -\hat{z} - (\mathcal{L}_{\mathcal{F}} \otimes I_n)e, \quad (19)$$

where $\hat{z}$ is governed by the dynamics

$$\dot{\hat{z}} = (I_F \otimes A)\hat{z}, \quad (20)$$

where (5), (8), (16), and the property of the Kronecker product $(A \otimes B)(C \otimes D) = (AC \otimes BD)$ were used.

IV. Convergence Analysis

In this section, the event-triggered controller designed in (4) is examined using a Lyapunov-based analysis. In addition to proving convergence of the error signal ε, the analysis also establishes a trigger condition associated with a trigger function that establishes when agents communicate state information.

To facilitate the subsequent convergence analysis, the event time t_k is explicitly defined below.

Definition 1. An event time t_k^i is defined as

$$t_k^i \triangleq \inf\left\{t > t_{k-1}^i \mid f_i(t) = 0\right\}, \; i \in \mathcal{V}_{\mathcal{F}} \tag{21}$$

for $k = 1, 2 \ldots,$ where $t_0^i = 0$, and $f_i(\cdot)$, denoted as $f_i(e_i(\cdot), \hat{z}_i(\cdot))$, is a trigger function defined as

$$f_i(e_i(t), \hat{z}_i(t)) \triangleq \|e_i(t)\| - \sqrt{\frac{\eta_i\left(\delta_1 - \frac{k_2}{\beta}\right)}{(k_1 + k_2\beta)}}\,\|\hat{z}_i(t)\|, \tag{22}$$

where $\eta_i \in \mathbb{R}_{>0}$ satisfying $0 < \eta_i \leq 1$ is a weighting term[1], and $\beta \in \mathbb{R}_{>0}$ is a positive constant satisfying

$$\beta > \frac{k_2}{\delta_1}. \tag{23}$$

In (22), $k_1, k_2 \in \mathbb{R}$ are positive constants defined as

$$k_1 \triangleq S_{\max}\left(\mathcal{L}_{\mathcal{F}}^3 \otimes \left(2PBB^TP\right) - \mathcal{L}_{\mathcal{F}}^2 \otimes \delta_1\right) \tag{24}$$

$$k_2 \triangleq \frac{1}{2} S_{\max}\left(\mathcal{L}_{\mathcal{F}} \otimes 2\delta_1 I_n - \mathcal{L}_{\mathcal{F}}^2 \otimes \left(2PBB^TP\right)\right), \tag{25}$$

where $\delta_1 \in \mathbb{R}_{>0}$ satisfies $0 < \delta_1 < \delta_{\min}$, and $S_{\max}(\cdot)$ denotes the maximum singular value of a matrix argument.

Theorem 1. *The controller designed in (4) ensures asymptotic containment control defined in (3) provided that the estimate $\hat{x}_i$ in (4) is updated at t_k^i defined in (21), for $i \in \mathcal{V}_{\mathcal{F}}$.*

Proof: Consider a candidate Lyapunov function $V : \mathbb{R}^{nF} \to \mathbb{R}$ defined as

$$V \triangleq \varepsilon^T (I_F \otimes P)\,\varepsilon, \tag{26}$$

where P is defined in (7). Using (6) and (17), the time derivative of (26) can be expressed as

$$\begin{aligned}\dot{V} =& \varepsilon^T\left[I_F \otimes \left(PA + A^TP\right) - \mathcal{L}_{\mathcal{F}} \otimes \left(2PBB^TP\right)\right]\varepsilon \\ &- e^T\left[\mathcal{L}_{\mathcal{F}}^2 \otimes \left(2PBB^TP\right)\right]\varepsilon.\end{aligned} \tag{27}$$

Using (7), (27) can be upper bounded by

$$\dot{V} \leq -\delta_{\min}\varepsilon^T\varepsilon - e^T\left[\mathcal{L}_{\mathcal{F}}^2 \otimes \left(2PBB^TP\right)\right]\varepsilon. \tag{28}$$

[1]η_i is a weighting term that has a trade-off between convergence performance and the size of inter-event interval. That is, moving η_i close to 1 can increase the inter-event interval, but the convergence performance is compromised, and vice versa.

Using (19), (28) can be upper bounded by

$$\begin{aligned}\dot{V} \leq& -\delta_1\left[\hat{z}^T\hat{z} + 2e^T(\mathcal{L}_{\mathcal{F}} \otimes I_n)\,\hat{z} + e^T\left(\mathcal{L}_{\mathcal{F}}^2 \otimes I_n\right)e\right] \\ &- e^T\left[\mathcal{L}_{\mathcal{F}}^2 \otimes \left(2PBB^TP\right)\right]\left[-\hat{z} - (\mathcal{L}_{\mathcal{F}} \otimes I_n)\,e\right] \\ &- \delta_2\varepsilon^T\varepsilon \\ \leq& -\delta_1\hat{z}^T\hat{z} - \delta_2\varepsilon^T\varepsilon \\ &+ e^T\left[\mathcal{L}_{\mathcal{F}}^3 \otimes \left(2PBB^TP\right) - \left(\mathcal{L}_{\mathcal{F}}^2 \otimes \delta_1\right)\right]e \\ &- e^T\left[(\mathcal{L}_{\mathcal{F}} \otimes 2\delta_1 I_n) - \mathcal{L}_{\mathcal{F}}^2 \otimes \left(2PBB^TP\right)\right]\hat{z},\end{aligned} \tag{29}$$

where $\delta_2 \in \mathbb{R}_{>0}$ satisfies $\delta_1 + \delta_2 = \delta_{\min}$. By using the inequality $x^Ty \leq \|x\|\,\|y\|$, (29) can be upper bounded as

$$\dot{V} \leq -\delta_1\|\hat{z}\|^2 + k_1\|e\|^2 + 2k_2\|e\|\,\|\hat{z}\| - \delta_2\varepsilon^T\varepsilon, \tag{30}$$

where k_1 and k_2 are defined in (24) and (25). Using the inequality $\|x\|\,\|y\| \leq \frac{\beta}{2}\|x\|^2 + \frac{1}{2\beta}\|y\|^2$, (30) can be upper bounded by

$$\begin{aligned}\dot{V} \leq& -\delta_1\|\hat{z}\|^2 + 2k_2\left(\frac{\beta}{2}\|e\|^2 + \frac{1}{2\beta}\|\hat{z}\|^2\right) + k_1\|e\|^2 \\ &- \delta_2\varepsilon^T\varepsilon \\ \leq& -\left(\delta_1 - \frac{k_2}{\beta}\right)\|\hat{z}\|^2 + (k_1 + k_2\beta)\|e\|^2 - \delta_2\varepsilon^T\varepsilon \\ \leq& -\sum_{i\in\mathcal{V}_{\mathcal{F}}}\left[\left(\delta_1 - \frac{k_2}{\beta}\right)\|\hat{z}_i\|^2 - (k_1 + k_2\beta)\|e_i\|^2\right] \\ &- \delta_2\varepsilon^T\varepsilon.\end{aligned} \tag{31}$$

In (31), two necessary conditions for $\dot{V}$ to be negative definite are to enforce $\delta_1 - \frac{k_2}{\beta} > 0$ and

$$\|e_i\|^2 \leq \frac{\eta_i\left(\delta_1 - \frac{k_2}{\beta}\right)}{(k_1 + k_2\beta)}\|\hat{z}_i\|^2, \tag{32}$$

which are satisfied provided the sufficient conditions in (21)-(23) are satisfied. Provided (32) is satisfied, then (31) can be rewritten as

$$\dot{V} \leq -\sum_{i\in\mathcal{V}_{\mathcal{F}}}(1 - \eta_i)\left(\delta_1 - \frac{k_2}{\beta}\right)\|\hat{z}_i\|^2 - \delta_2\varepsilon^T\varepsilon, \tag{33}$$

$$\leq -\delta_2\varepsilon^T\varepsilon. \tag{34}$$

The linear differential inequality resulting from (26) and (34) can be solved to conclude that

$$\|\varepsilon\| \leq \|\varepsilon(t_0)\|\,e^{-\gamma(t-t_0)},$$

where $\gamma \in \mathbb{R}_{>0}$ is a positive constant. Therefore, from (13) the convergence of ε implies (3). ■

Remark 1. The Riccati inequaty defined in (7) and used in (28) is developed to facilitate the stability analysis.

V. Minimum inter-event interval

To show the proposed trigger functions in Definition 1 do not lead to Zeno behavior, it is sufficient to find a positive lower bound for the inter-event interval. To facilitate subsequent analysis, two constants $\bar{c}_0,\, \bar{c}_1 \in \mathbb{R}_{>0}$ are defined as

$$\bar{c}_0 \triangleq S_{\max}\left(I_F \otimes BK\right) \tag{35}$$

$$\bar{c}_1 \triangleq S_{\max}\left(\left(I_F \otimes A\right) + \left(\mathcal{L}_{\mathcal{F}} \otimes BK\right)\right) + S_{\max}\left(\mathcal{L}_{\mathcal{F}} \otimes BK\right) + S_{\max}\left(I_F \otimes A\right). \tag{36}$$

Theorem 2. *The event time defined in (21) ensures that there exists an agent $h \in \mathcal{V}_{\mathcal{F}}$ such that its minimum inter-event interval $\tau \in \mathbb{R}$ is lower bounded by*

$$\tau \geq \frac{1}{\max\left\{\bar{c}_0,\, \bar{c}_1\right\}} \ln\left(\frac{1}{F}\sqrt{\frac{\eta_h\left(\delta_1 - \frac{k_2}{\beta}\right)}{\left(k_1 + k_2\beta\right)}} + 1\right), \tag{37}$$

where h is an agent that satisfies

$$h = \arg\max_{i \in \mathcal{V}} \sup_{t \in \mathbb{R}_{\geq 0}} \left\|\hat{z}_i\right\|,$$

and F is the number of follower agents defined in Section II-A.

Proof: For further details, please see [19, Theorem 2]. ■

Remark 2. This lower bound implies that Zeno behaviors can be excluded. However, there is a trade-off between the minimum inter-event interval and the error convergence rate. The lower bound in (37) can be increased by selecting a higher η_h, but this increase results in a slower convergence due to the fact that $\dot{V}$ in (33) becomes less negative. Every agent has the freedom to adjust its η_h to make the minimum inter-event interval flexible.

VI. Discussion

A decentralized event-triggered control scheme for the containment control problem is developed to reduce communication frequencies between neighboring agents while ensuring stability of the system. The estimate-based controller along with the decentralized trigger function reduces the number of inter-agent communication events, during which no communication is required. A Lyapunov-based analysis indicates that the networked system achieves asymptotic containment control under this event-triggered control scheme where the trigger condition does not exhibit Zeno behavior.

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